proparty of limit

$$
\begin{aligned}
1-\lim_{x\to}[f(x)\pm g(x)] &= \lim_{x\to} f(x)\pm \lim_{x\to} g(x)\\
2-\lim_{x\to}[f(x)\cdot g(x)] &= \lim_{x\to} f(x)\cdot \lim_{x\to} g(x)\\
3-\lim_{x\to}[\frac{f(x)}{g(x)}] &= \frac{\lim_{x\to} f(x)}{\lim_{x\to} g(x)}\\
4-\lim_{x\to}[cf(x)] &= c\lim_{x\to} f(x)\\
5-\lim_{x\to}[f(x)]^n &= [\lim_{x\to} f(x)]^n, n \text{is an integer \ number}\\
6-\lim_{x\to}[f(x)]^n &= [\lim_{x\to} f(x)]^n, n \text{ is an rational number if n is even} \Rightarrow \lim_{x\to} f(x)>0
\end{aligned}
$$

or we can written as

$$\lim_{x\to} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to} f(x)}, \text{if } n \text{ is even} \Rightarrow \lim_{x\to} f(x)>0$$

$$7-\lim_{x\to} a^{f(x)} = a^{\lim_{x\to} f(x)}$$

simmilar,

$$\lim_{x\to} e^{f(x)} = e^{\lim_{x\to} f(x)}$$

$$8-\lim_{x\to}[\log_a[f(x)]] = [\log_a[\lim_{x\to} f(x)]] \text{ such that } \lim_{x\to} f(x)>0$$

simillar,

$$\lim_{x\to}[\ln[f(x)]] = [\ln[\lim_{x\to} f(x)]] \text{ such that } \lim_{x\to} f(x)>0$$

$$
\begin{aligned}
9-\lim_{x\to}[\sin[f(x)] &= [\sin[\lim_{x\to} f(x)],\\
\lim_{x\to}[\cos[f(x)] &= [\cos[\lim_{x\to} f(x)],\\
\lim_{x\to}[\tan[f(x)] &= [\tan[\lim_{x\to} f(x)],\\
\lim_{x\to}[\cot[f(x)] &= [\cot[\lim_{x\to} f(x)],\\
\lim_{x\to}[\sec[f(x)] &= [\sec[\lim_{x\to} f(x)],\\
\lim_{x\to}[\csc[f(x)] &= [\csc[\lim_{x\to} f(x)].
\end{aligned}
$$

simillar for the invers trigonometric function.

The Sandwich Theorem:
if the function $f(x)$ satisfy the condition

$$g(x) \leq f(x) \leq h(x)$$

and

$$\lim_{x\to} g(x) = \lim_{x\to} h(x) = L$$

Then,

$$\lim_{x\to} f(x) = L$$

To find the limit of function:

1- constant finction:

limit approchis to point a

$$\lim_{x\to a} c = c$$

One side limit

$$\begin{aligned}\lim_{x\to a^+} c &= c\\ \lim_{x\to a^-} c &= c\end{aligned}$$

limit approches to $\pm\infty$

$$\lim_{x\to\pm\infty} c = c$$

2- Polinomal function $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_\circ$

limit approchis to point a

$$\lim_{x\to a} f(x) = f(a)$$

Direct compensation

-One side limit

$$\begin{aligned}\lim_{x\to a^+} f(x) &= f(a)\\ \lim_{x\to a^-} f(x) &= f(a)\end{aligned}$$

Direct compensation

-limit x approches to $\pm\infty$

$$\begin{aligned}\lim_{x\to+\infty} x^n &= \infty,\ n \text{ even},\\ \lim_{x\to-\infty} x^n &= \infty,\ n \text{ even},\\ \lim_{x\to+\infty} x^n &= \infty,\ n \text{ odd},\\ \lim_{x\to-\infty} x^n &= -\infty,\ n \text{ odd}.\end{aligned}$$

$$\lim_{x\to\pm\infty} a_n x^n + a_{n-1}x^{n-1} + ... + a_\circ = \lim_{x\to\pm\infty} a_n x^n$$

3- rational function f(x)=$\frac{p(x)}{q(x)} = \frac{a_n x^n + a_{n-1}x^{n-1}+...+a_\circ}{b_m x^m + b_{m-1}x^{m-1}+...+a_\circ}$

- limit x approches to a point a

$$\lim_{x\to a} f(x) = f(a) = \frac{p(a)}{q(a)}$$

Direct compensation

-One side limit

$$\lim_{x\to a^+} f(x) = f(a) = \frac{p(a)}{q(a)}$$
$$\lim_{x\to a^-} f(x) = f(a) = \frac{p(a)}{q(a)}$$

Direct compensation

-limit x approches to $\pm\infty$

$$\lim_{x\to +\infty} \frac{1}{x} = \infty$$
$$\lim_{x\to -\infty} \frac{1}{x} = -\infty$$
$$\lim_{x\to +\infty} \frac{1}{x^2} = \infty$$
$$\lim_{x\to -\infty} \frac{1}{x^2} = \infty$$
$$\lim_{x\to \pm\infty} \frac{1}{x^n} = +\infty, n \text{ is an even}$$
$$\lim_{x\to +\infty} \frac{1}{x^n} = +\infty, n \text{ is an odd}$$
$$\lim_{x\to -\infty} \frac{1}{x^n} = -\infty, n \text{ is an odd}$$

if $n = m$ (degree of $p(x)$ = degree of $g(x)$)$\Rightarrow$

$$\lim_{x\to \pm\infty} \frac{a_n x^n + a_{n-1}x^{n-1}+...+a_\circ}{b_m x^m + b_{m-1}x^{m-1}+...+a_\circ} = \frac{a_n}{b_n}$$

If $n < m$ (degree of $p(x) <$ degree of $g(x)$) $\Rightarrow$

$$\lim_{x\to \pm\infty} \frac{a_n x^n + a_{n-1}x^{n-1}+...+a_\circ}{b_m x^m + b_{m-1}x^{m-1}+...+a_\circ} = 0$$

If $n < m$ (degree of $p(x)$ $>$ degree of $g(x)$) $\Rightarrow$

$$\lim_{x\to \pm\infty} \frac{a_n x^n + a_{n-1}x^{n-1}+...+a_\circ}{b_m x^m + b_{m-1}x^{m-1}+...+a_\circ} = \pm\infty$$

to know it is $+\infty$ or $-\infty$ test the signe of $\frac{a_n x^n}{b_m x^m}$

4- root function $\sqrt[n]{f(x)}$

If n is odd number

- limit x approches to a point a

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{f(a)}$$

Direct compensation

-One side limit

$$\lim_{x \to a^+} \sqrt[n]{f(x)} = \sqrt[n]{f(a)}$$
$$\lim_{x \to a^-} \sqrt[n]{f(x)} = \sqrt[n]{f(a)}$$

Direct compensation

If n is even number

- limit x approches to a point a

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{f(a)}$$

Direct compensation
if under the root positive $\Rightarrow$ the solution is end and have value
if under the root 0 or nigative $\Rightarrow$ the solution is D.N.E. (dose not exist)

-One side limit

$$\lim_{x \to a^+} \sqrt[n]{f(x)} = \sqrt[n]{f(a)}$$
$$\lim_{x \to a^-} \sqrt[n]{f(x)} = \sqrt[n]{f(a)}$$

Direct compensation
if under the root positive $\Rightarrow$ the solution is end and have value

if under the root 0 $\Rightarrow$find the domain
if the graph from the right $\Rightarrow \lim_{x \to a^+} \sqrt[n]{f(x)} = 0$ and $\lim_{x \to a^-} \sqrt[n]{f(x)}$ D.N.E.
if the graph from the left $\Rightarrow \lim_{x \to a^+} \sqrt[n]{f(x)} =$D.N.E. and $\lim_{x \to a^-} \sqrt[n]{f(x)} = 0$
if under the root nigative $\Rightarrow$ the solution is D.N.E. (dose not exist)

-limit x approches to $\pm\infty$

divaided by x $^{\text{largest}}$
if $x \to +\infty$ then the result is positive
if $x \to -\infty$ then the result is nigative

5- Exponitial function $f(x) = a^x$ where $a > 1$ or $f(x) = e^x$

Note that:

$$\begin{aligned} a^0 &= 1 \\ a^\infty &= \infty \\ a^{-\infty} &= 0 \end{aligned}$$

simmilar,

$$\begin{aligned} e^0 &= 1 \\ e^\infty &= \infty \\ e^{-\infty} &= 0 \end{aligned}$$

limit approchis to point a

$$\lim_{x \to a} f(x) = f(a)$$

Direct compensation

-One side limit

$$\begin{aligned} \lim_{x \to a^+} f(x) &= f(a) \\ \lim_{x \to a^-} f(x) &= f(a) \end{aligned}$$

Direct compensation

-limit x approches to $\pm\infty$

Direct compensation

6- logarithmic finction $f(x) = \log_a x$ or $f(x) = \ln x$

Note:

$$\begin{aligned} \ln 1 &= 0 \\ \ln \infty &= \infty \\ \ln -\infty &= \text{D.N.E.} \end{aligned}$$

limit approchis to point a

$$\lim_{x \to a} f(x) = f(a)$$

Direct compensation

if inside the lograthmic function positive $\Rightarrow$ the solution is end and have value

if under the root 0 or nigative $\Rightarrow$ the solution is D.N.E. (dose not exist)

-One side limit

$$\lim_{x\to a^+} f(x) = f(a)$$
$$\lim_{x\to a^-} f(x) = f(a)$$

Direct compensation

if inside the lograthmic function positive $\Rightarrow$ the solution is end and have value

if under the root 0 $\Rightarrow$find the domain

if the graph from the right $\Rightarrow \lim_{x\to a^+} f(x) = \ln 0 = -\infty$ or $\lim_{x\to a^+} f(x) = \log 0 = -\infty$

and $\lim_{x\to a^-} f(x)$ D.N.E.

if the graph from the left $\Rightarrow \lim_{x\to a^+} f(x)$ D.N.E.

and $\lim_{x\to a^-} f(x) = \ln 0 = -\infty$ or $\lim_{x\to a^-} f(x) = \log 0 = -\infty$

if under the root nigative $\Rightarrow$ the solution is D.N.E. (dose not exist)

-limit x approches to $+\infty$

Direct compensation

Continouty

$f(x)$ is continous at a if and only if satisfy

1- $f(a)$ defined
2- $\lim_{x\to a} f(x)$ exist
3- $f(a) = \lim_{x\to a} f(x)$

$f(x)$ is continous from right of a if and only if satisfy

1- $f(a)$ defined
2- $\lim_{x\to a^+} f(x)$ exist
3- $f(a) = \lim_{x\to a^+} f(x)$

$f(x)$ is continous from left of a if and only if satisfy

1- $f(a)$ defined
2- $\lim_{x\to a^-} f(x)$ exist
3- $f(a) = \lim_{x\to a^-} f(x)$

all function are continous on its domaine

[algebric, polynomial, rational, root, exponitiol, logarithmic and trigonmetric function]